

NASA-TM-78803 19790006688

Mikhailov Stability Criterion for Time-Delayed Systems

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JANUARY 1979

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National Aeronautics
and Space Administration

**Scientific and Technical
Information Office**

1979

SUMMARY

The valid and invalid application of the Mikhailov criterion to linear, time-invariant systems with time delays is discussed. The Mikhailov criterion is a graphical procedure which was initially developed to examine the stability of linear, time-invariant systems with no time delays. For these systems, there are two equivalent formulations of the criterion. Attempting to apply the second formulation when there are time delays in the systems can lead to erroneous results, as shown by an example. However, the first formulation remains valid for time-delayed systems of the retarded type, with the understanding that the Mikhailov curve need not necessarily always rotate in the counterclockwise direction for a stable system. At present, application of the Mikhailov criterion to neutral systems has not been justified.

INTRODUCTION

The Mikhailov criterion is used to examine the stability of dynamical systems that are described by linear ordinary differential equations with constant coefficients. Kashiwagi (ref. 1) indicates that this criterion can be used with only limited success when there are constant time delays in the system. Yet recently, Chen and Tsay (ref. 2) have shown that a modified Nyquist criterion, which is similar to the Mikhailov criterion, does have valid application to time-delayed systems. Why is there a discrepancy between the earlier and recent works? The purpose of this paper is to remove the confusion related to the valid application of the Mikhailov criterion to time-delayed systems.

SYMBOLS

| | |
|---------------|-----------------------------|
| A, A_j, B_j | real constants |
| i | imaginary unit, $\sqrt{-1}$ |
| j | integer index |
| k | real number |

| | |
|------------------------------------|---|
| $L(s)$ | characteristic polynomial for system with no time delays, or characteristic quasi-polynomial for systems with time delays |
| $L_1(s), L_2(s)$ | parts of $L(s)$ shown in equations (42) and (43) |
| M | number of roots of $L(s)$ with positive real parts |
| N | order of system with zero time delays |
| s | complex variable |
| s_j | j th root of $L(s) = 0$ |
| $X(\omega)$ | real part of $L(i\omega)$ |
| $Y(\omega)$ | imaginary part of $L(i\omega)$ |
| γ | angle in figure 2 |
| ξ | damping parameter |
| $\theta(\omega), \theta_j(\omega)$ | change in arguments of $L(i\omega)$ and $i\omega - s_j$, respectively, as ω increases from zero |
| $\tilde{\theta}, \tilde{\theta}_j$ | total change in arguments of $L(i\omega)$ and $i\omega - s_j$, respectively, as ω varies increasingly from zero to infinity |
| σ | real part of s |
| σ_∞ | asymptote of real part of large modulus roots of $L(s)$ |
| τ, τ_j | constant real-time delays |
| ω | imaginary part of s |

ANALYSIS

By referring to the complex geometry involved, Popov (ref. 3) gives insight into the Mikhailov criterion for a system with no time delays. Kashiwagi used Popov as a basic

reference. Hence, in order to appreciate the earlier thoughts on the Mikhailov criterion, it is necessary to first examine the criterion for no time delays as viewed by Popov.

Mikhailov Criterion (No Delays)

Let the characteristic polynomial of a linear system be expressed as

$$L(s) = A_0 s^N + A_1 s^{N-1} + \dots + A_{N-1} s + A_N \quad (A_0 > 0) \quad (1)$$

with real coefficients, or in factored form as

$$L(s) = A_0 (s - s_1) (s - s_2) \dots (s - s_N) \quad (2)$$

where the complex roots appear in complex conjugate pairs. With $s = i\omega$, equation (1) can be written as

$$L(i\omega) = X(\omega) + iY(\omega) \quad (3)$$

where

$$\left. \begin{aligned} X(\omega) &= A_N - A_{N-2}\omega^2 + A_{N-4}\omega^4 - \dots \\ Y(\omega) &= A_{N-1}\omega - A_{N-3}\omega^3 + \dots \end{aligned} \right\} \quad (4)$$

and equation (2) becomes

$$L(i\omega) = A_0 (i\omega - s_1) (i\omega - s_2) \dots (i\omega - s_N) \quad (5)$$

Now consider the plot of $L(i\omega)$ in the complex plane as ω is increased from 0 to ∞ , as depicted hypothetically in figure 1. This is the so-called Mikhailov curve. The argument of $L(i\omega)$ is simply the sum of the arguments of the linear factors of $L(i\omega)$ in equation (5), that is,

$$\arg L(i\omega) = \sum_{j=1}^N \arg (i\omega - s_j) \quad (6)$$

Let $\theta(\omega)$ and $\theta_j(\omega)$ denote the changes in the arguments of $L(i\omega)$ and $i\omega - s_j$, respectively, as ω increases from zero. Then,

$$\theta(\omega) = \sum_{j=1}^N \theta_j(\omega) \quad (7)$$

(The argument of the real number $A_0 > 0$ is zero.) Furthermore, if $\tilde{\theta}$ and $\tilde{\theta}_j$ denote the total change in the arguments of $L(i\omega)$ and $i\omega - s_j$, respectively, as ω varies from 0 to ∞ , then

$$\tilde{\theta} = \sum_{j=1}^N \tilde{\theta}_j \quad (8)$$

Consider the following two special cases to gain insight into the contributions of $\tilde{\theta}_j$ to $\tilde{\theta}$ in equation (8).

Negative real root.- The change in the argument of the factor $i\omega - s_1$, when s_1 is a negative real number, is examined by using figure 2(a). It is geometrically clear that as ω varies from zero to infinity

$$\tilde{\theta}_1 = \lim_{\omega \rightarrow \infty} \theta_1(\omega) = \frac{\pi}{2} \quad (9)$$

Complex conjugate roots.- Consider the angular change in the arguments of the two factors $(i\omega - s_2)$ and $(i\omega - s_3)$, when s_2 and s_3 are complex conjugate roots with negative real parts, as indicated in figure 2(b). Geometrically, it can be seen that for $\omega = 0$, the argument of the complex vector $i\omega - s_2$ is $-\gamma$. As ω increases from zero to infinity, the change in the argument is

$$\tilde{\theta}_2 = \lim_{\omega \rightarrow \infty} \theta_2(\omega) = \frac{\pi}{2} + \gamma \quad (10)$$

Similarly, for the vector $i\omega - s_3$,

$$\tilde{\theta}_3 = \lim_{\omega \rightarrow \infty} \theta_3(\omega) = \frac{\pi}{2} - \gamma \quad (11)$$

Hence,

$$\tilde{\theta}_2 + \tilde{\theta}_3 = \frac{\pi}{2} + \frac{\pi}{2} \quad (12)$$

It is indicated by equations (9) and (12) that each root of $L(s)$ with a negative real part contributes $\frac{\pi}{2}$ to the argument of $L(i\omega)$ in equation (8). By similar geometry, it can be seen that any root of $L(s)$ with a positive real part will result in an argument change in $L(i\omega)$ of $-\frac{\pi}{2}$ as ω goes from 0 to ∞ . Popov (ref. 3) also discusses the effects of a zero root, purely imaginary roots, and an infinite root, but these cases are not presented here.

Suppose there are M roots of $L(s)$ in equation (1) with positive real parts, and $N - M$ roots with negative real parts; then it is not difficult to infer geometrically that

$$\tilde{\theta} = (N - M)\frac{\pi}{2} - M\frac{\pi}{2} = (N - 2M)\frac{\pi}{2} \quad (13)$$

In order for the systems under consideration to be stable, all roots of $L(s)$ must have negative real parts. In this respect, Popov (ref. 3) writes: "For stability of an n th-order linear system it is necessary and sufficient that the Mikhailov curve plotted for the characteristic equation of the given system pass through n quadrants in succession counterclockwise, circling the origin of coordinates." Thus, since $n = N$ in this paper, $L(i\omega)$ completes a rotation by the angle

$$\tilde{\theta} = N\frac{\pi}{2} \quad (14)$$

which is equation (13) with $M = 0$ (no roots with positive real parts). This is the first formulation of the Mikhailov criterion.

The roots s_1 , s_2 , and s_3 of $L(s)$ are shown in figure 2 and have negative real parts. The associated vectors $i\omega - s_1$, $i\omega - s_2$, and $i\omega - s_3$ always rotate counterclockwise as ω increases; or equivalently, θ_1 , θ_2 , and θ_3 always increase. Therefore, from equation (7), if all roots of $L(s)$ have negative real parts, it is not possible for the vector $L(i\omega)$ to ever rotate by any amount in the clockwise direction; that is, the vector $L(i\omega)$ continually rotates in the counterclockwise direction as ω increases from 0 to ∞ , and $\theta(\omega)$ approaches $N\frac{\pi}{2}$ (eq. (14)). This leads to the second formulation of the Mikhailov criterion, which states that a necessary and sufficient condition for a linear, time-invariant system with no time delays to be stable (characteristic roots with negative real parts) is that the real part, $X(\omega)$, and the imaginary part, $Y(\omega)$, of $L(i\omega)$ in equation (3) alternately vanish a finite number of times.

Transcendental Characteristic Equation

Previous works (refs. 3 and 4, for example) interpret the Mikhailov criterion in terms of a polynomial characteristic equation. Difficulty occurs in trying to apply this interpretation to systems with time delays (ref. 1).

Suppose the system has time delays so that the characteristic function is, for example, the transcendental characteristic quasi-polynomial

$$L(s) = s^2 + 2\zeta s e^{-\tau s} + 1 \quad (15)$$

where τ is a constant time delay. Unlike equation (1) which has exactly N roots, equation (15) has an infinite number of roots because of the exponential term introduced by the time delay.

For illustration, let $\tau = 6.2$ and $\zeta = 0.2$ in equation (15). In this specific case, all the roots of equation (15) have negative real parts, making the system stable (ref. 1).

The Mikhailov curve corresponding to equation (15) is shown in figure 3. Notice that the curve moves in both the clockwise and counterclockwise directions even though all the roots have negative real parts. This could never happen if $L(s)$ were a polynomial with negative real parts. To approximate the curve in figure 3 very closely with a polynomial would require that the approximating polynomial have an unstable root (positive real part) in order that the approximating curve ever move in a clockwise direction over any portion of the solution.

Clearly, from figure 3, $X(\omega)$ and $Y(\omega)$ do not alternately vanish; and, consequently (by counterexample), the second formulation of the Mikhailov criterion definitely does not apply in general. However, as noted in the section entitled "Mikhailov Criterion for Delayed Systems," with proper interpretation, the first formulation of the Mikhailov criterion can be applied.

Mikhailov Criterion for Delayed Systems

Recently, Chen and Tsay (ref. 2) derived a modified Nyquist stability criterion which, in the context of this paper, is similar to the first formulation of the Mikhailov criterion. The criterion was shown to hold for characteristic functions $L(s)$ for which:

- (1) $L(s)$ has no purely imaginary roots.
- (2) $L^*(s) = L(s^*)$, where $*$ means complex conjugate.
- (3) $L(s)$ acts like As^{-k} as $s \rightarrow \infty$ in the right half of the s -plane; that is,

$$\lim_{s \rightarrow \infty} \frac{L(s)}{s^{-k}} = A \neq 0 \quad (\sigma \geq 0) \quad (16)$$

where A is a nonzero constant and k is an integer.

Chen and Tsay (ref. 2) show that, as ω increases from 0 to ∞ , the vector $L(i\omega)$ rotates by the amount

$$\tilde{\theta} = (-k - 2M) \frac{\pi}{2} \quad (17)$$

where M is the number of roots with positive real parts.

Equation (17) indicates a stable system if and only if $M = 0$; that is

$$\tilde{\theta} = -k \frac{\pi}{2} \quad (18)$$

It is important to note that there is no requirement that $\tilde{\theta}$ result only from counterclockwise rotations.

With respect to the characteristic polynomial in equation (1), $k = -N$ in equation (16), and equation (18) becomes

$$\tilde{\theta} = N \frac{\pi}{2} \quad (19)$$

which is the same as equation (14). Now, from Popov's view of the Mikhailov criterion, it is clear that $\tilde{\theta}$ in equation (19) will, indeed, consist only of counterclockwise rotations. However, this is not explicit in the derivation (ref. 2) of equation (18).

Retarded systems. - Consider a system with a characteristic quasi-polynomial of the form

$$L(s) = \sum_{j=0}^N A_j s^j + \sum_{j=0}^{N-1} B_j s^j e^{-\tau_j s} \quad (20)$$

where $A_N \neq 0$ and $\tau_j \geq 0$ (ref. 5). The dominant term in equation (20) as $s \rightarrow \infty$ is $A_N s^N$. To show this, consider the magnitude of the ratio of any remaining term in equation (20) relative to $A_N s^N$. Thus, note that for $j \neq N$,

$$\lim_{|s| \rightarrow \infty} \left| \frac{A_j s^j}{A_N s^N} \right| = \lim_{|s| \rightarrow \infty} \left| \frac{A_j}{A_N} \right| \cdot \frac{1}{|s|^{N-j}} = 0 \quad (21)$$

since $N - j > 0$. Also,

$$\lim_{|s| \rightarrow \infty} \left| \frac{B_j s^j}{A_N s^N} e^{-\tau_j s} \right| = \lim_{|s| \rightarrow \infty} \left| \frac{B_j}{A_N} \right| \cdot \frac{e^{-\tau_j \sigma}}{|s|^{N-j}} = 0 \quad (22)$$

where σ is the real part of s . Hence,

$$\lim_{s \rightarrow \infty} \frac{L(s)}{s^N} = A_N \neq 0 \quad (\sigma \geq 0) \quad (23)$$

for the retarded system, and it follows from equation (16) that $k = -N$. Therefore, from equation (18), the system is stable if and only if

$$\tilde{\theta} = N \frac{\pi}{2} \quad (24)$$

For example, suppose the retarded system has the characteristic quasi-polynomial in equation (15). Then, equation (24) indicates that $L(i\omega)$, which is shown in figure 3, should rotate by the angle $2\left(\frac{\pi}{2}\right) = \pi$ for stability. The real part of $L(i\omega)$ in figure 3 is increasing much faster than its imaginary part; therefore, the change in the argument of $L(i\omega)$ as ω varies from 0 to ∞ appears to be approaching π . This conjecture is verified algebraically as follows. With $s = i\omega$, equation (15) becomes

$$L(i\omega) = \left(1 - \omega^2 + 2\zeta\omega \sin \omega\tau\right) + (2\zeta\omega \cos \omega\tau)i \quad (25)$$

The real part of $L(i\omega)$ in equation (25) is

$$X(\omega) = 1 - \omega^2 + 2\zeta\omega \sin \omega\tau \quad (26)$$

Notice in equation (26) that if

$$\omega^2 > 1 + 2\zeta\omega \sin \omega\tau \quad (27)$$

then $X(\omega) < 0$. Certainly, equation (27) will be true if

$$\omega^2 > 1 + 2\xi\omega \quad (28)$$

The right-hand side of equation (28) dominates the left side as ω approaches 0. However, as ω increases, the left side will eventually dominate the right, and equation (28) will be valid. Equation (28) is valid for all $\omega > \omega_m$, where ω_m is the largest positive real root of the polynomial equation

$$\omega^2 - 2\xi\omega - 1 = 0 \quad (29)$$

and is given by

$$\omega_m = \xi + \sqrt{\xi^2 + 1} \quad (30)$$

For the specific example being considered, $\xi = 0.2$ and $\tau = 6.2$. Thus, equations (26) and (30) become, respectively,

$$X(\omega) = 1 - \omega^2 + 0.4\omega \sin 6.2\omega \quad (31)$$

$$\omega_m = 1.2 \quad (32)$$

The real and imaginary parts of $L(i\omega)$ are parametric equations in ω , with ω increasing along the Mikhailov curve in figure 3. After ω increases to $\omega = \omega_m = 1.2$, the corresponding $X(\omega)$ value is $X(\omega_m) = 0.056$. Thereafter, $\omega > \omega_m$ and corresponding values of $X(\omega)$ will remain negative. In other words, if continued, the Mikhailov curve in figure 3 will remain in the left half plane (second and third quadrants). Since the curve in figure 3 does not encircle the origin, the change in the argument of $L(i\omega)$ as ω varies from 0 to ∞ lies in the interval

$$\frac{\pi}{2} < \theta < \frac{3\pi}{2} \quad (33)$$

Since the argument of $L(i\omega)$ in figure 3 is zero at $\omega = 0$, $\theta(\omega) = \arg [L(i\omega)]$. Hence,

$$\cos \theta(\omega) = \frac{1 - \omega^2 + 2\xi\omega \sin \omega\tau}{\sqrt{(2\xi\omega \cos \omega\tau)^2 + (1 - \omega^2 + 2\xi\omega \sin \omega\tau)^2}} \quad (34)$$

from which it follows that

$$\lim_{\omega \rightarrow \infty} \cos \theta(\omega) = -1 \quad (35)$$

Equation (35), along with inequality (33), implies that $\theta(\omega) \rightarrow \pi$ as $\omega \rightarrow \infty$, so that the system is stable for $\tau = 6.2$ and $\xi = 0.2$.

Kashiwagi (ref. 1) was dealing with retarded systems when he stated that the Mikhailov criterion could be used with only limited success when there are constant time delays in the system. He draws this conclusion from the second formulation of the Mikhailov criterion, which is associated with characteristic polynomials. Indeed, as substantiated by the example in this paper, the second formulation does not hold in general for retarded systems; however, the first formulation remains valid if it is recognized that the Mikhailov curve need not always rotate counterclockwise. The only requirement is that the curve complete a rotation of $N\frac{\pi}{2}$ (eq. (24)). This appears to be the basic reason for the discrepancy between earlier and recent works.

The first formulation of the Mikhailov criterion can be used to determine if a retarded system is stable for a fixed set of time delays. In programing the criterion on a digital computer, it is not necessary to express the characteristic equation in simplified form, for example, like equations (1) or (3).

The Mikhailov criterion cannot be used readily to generate stability boundaries. In this respect, however, parameter-plane methods (refs. 6 and 7, for example) are perhaps more appropriate.

Neutral systems.— A system having the highest ordered derivative to appear both with and without a time delay is called a neutral system. A class of neutral systems has the following characteristic quasi-polynomial (ref. 5):

$$L(s) = \sum_{j=0}^N A_j s^j + \sum_{j=0}^N B_j s^j e^{-\tau_j s} \quad (36)$$

where $A_N \neq 0$, $B_N \neq 0$, $\tau_j \geq 0$ for $j \neq N$, and $\tau_N > 0$. (For $A_N \neq 0$ and $B_N = 0$, the neutral system reduces to a retarded system, provided there is at least one $\tau_j \neq 0$ for $j \neq N$.) These systems are stable if all the roots of equation (36) have negative real parts and are not asymptotic to the imaginary axis (ref. 8). The stability is uncertain if the roots are asymptotic to the imaginary axis. However, it appears that a system with such roots would be too lightly damped. In addition, because of the uncertainty in system coefficients, it seems desirable not to have a root too close to the imaginary axis. Within the

context of this paper, this condition is simply considered unsatisfactory. The system is unstable if any root has a positive real part. (See ref. 9.) A practical example of a neutral system is examined in reference 10.

The stability of neutral systems has been determined recently by the extended τ -decomposition method (refs. 11 and 12). In this section, the possibility of also applying the first formulation of the Mikhailov criterion to the class of neutral systems is examined.

The characteristic equation for the neutral system is $L(s) = 0$, which can be expressed as

$$A_N s^N = B_N s^N e^{-\tau_N s} + \sum_{j=0}^{N-1} (A_j + B_j e^{-\tau_j s}) s^j \quad (37)$$

All roots of equation (37) must satisfy the relation

$$\left| |A_N| - |B_N| e^{-\tau_N \sigma} \right| |s|^N \leq \sum_{j=0}^{N-1} \left(|A_j| + |B_j| e^{-\tau_j \sigma} \right) |s|^j \quad (38)$$

which is obtained by applying properties of the absolute value to equation (37). Implicit in inequality (38) is the requirement that as $|s| \rightarrow \infty$ (i.e., the modulus of the roots increases without bound),

$$\lim_{|s| \rightarrow \infty} \left(|A_N| - |B_N| e^{-\tau_N \sigma} \right) = 0 \quad (39)$$

From equation (39), σ becomes arbitrarily close to

$$\sigma_\infty = -\frac{1}{\tau_N} \ln \left| \frac{A_N}{B_N} \right| \quad (40)$$

That is, the real parts of the characteristic roots form a sequence which approaches σ_∞ . As long as $\sigma_\infty \neq 0$, the roots will not be asymptotic to the imaginary axis. However, it still remains to be determined whether there are any roots with positive real parts (unstable).

Three distinct cases are of interest, namely:

$$\text{Case 1: } \frac{A_N}{B_N} < 1 \quad \text{or} \quad \sigma_\infty > 0 \quad (\text{unstable})$$

$$\text{Case 2: } \frac{A_N}{B_N} = 1 \quad \text{or} \quad \sigma_\infty = 0 \quad (\text{unsatisfactory})$$

$$\text{Case 3: } \frac{A_N}{B_N} > 1 \quad \text{or} \quad \sigma_\infty < 0 \quad (\text{either stable or unstable})$$

In equation (40), $\tau_N > 0$. Thus, for Case 1, $\sigma_\infty > 0$ and the neutral system is unstable. For Case 2, $\sigma_\infty = 0$, the roots are located too close to the imaginary axis, and the system is considered unsatisfactory. Finally, in Case 3, $\sigma_\infty < 0$ and the neutral system is either stable or unstable, being stable if and only if all roots have negative real parts.

The remainder of this paper is concerned with Case 3 of the neutral system. If it could be shown that $L(s)$ in equation (36) satisfies equation (16), then the previous results of Chen and Tsay (ref. 2) could be used to justify the application of the Mikhailov criterion. However, as the following analysis shows, this justification was not possible.

For convenience of discussion, define

$$L(s) = L_1(s) + L_2(s) \quad (41)$$

where

$$L_1(s) = \left(A_N + B_N e^{-\tau_N s} \right) s^N \quad (42)$$

and

$$L_2(s) = \sum_{j=0}^{N-1} \left(A_j + B_j e^{-\tau_j s} \right) s^j \quad (43)$$

It will be shown that $L_1(s)$ is the dominant term of $L(s)$ as $s \rightarrow \infty$ in the right half of the s -plane ($\sigma \geq 0$). Inequality relationships involving the magnitudes of $L_1(s)$ and $L_2(s)$ are obtained from equations (42) and (43) as

$$|L_1(s)| \geq \left| |A_N| - |B_N| e^{-\tau_N \sigma} \right| |s|^N \quad (44)$$

and

$$|L_2(s)| \leq \sum_{j=0}^{N-1} \left(|A_j| + |B_j| e^{-\tau_j \sigma} \right) |s|^j \quad (45)$$

If the coefficient of $|s|^N$ in equation (44) is bounded away from zero (greater than some positive number) and the coefficient of each $|s|^j$ in equation (45) is less than some finite positive number, then eventually $|L_1(s)|$ will dominate $|L_2(s)|$ as $s \rightarrow \infty$ in the right half of the s -plane. There is no conflict with equation (39) because all but finitely many roots of the characteristic equation are in the left half of the s -plane for Case 3.

For $0 < \sigma < \infty$ and since $|A_N| > |B_N|$, the coefficient on the $|s|^N$ term is bounded away from zero as

$$0 < |A_N| - |B_N| \leq \left| |A_N| - |B_N| e^{-\tau_N \sigma} \right| \quad (46)$$

so it is greater than the positive finite number $|A_N| - |B_N|$.

For $0 < \sigma < \infty$, the coefficient of the $|s|^j$ term is bounded above as

$$|A_j| + |B_j| e^{-\tau_j \sigma} \leq |A_j| + |B_j| \quad (47)$$

so it is less than the finite number $|A_j| + |B_j|$. Hence, the dominant term of $L(s)$ is $L_1(s)$ as $s \rightarrow \infty$ with $\sigma \geq 0$, or

$$\lim_{s \rightarrow \infty} \frac{L(s)}{s^N (A_N + B_N e^{-\tau_N \sigma})} = 1 \quad (\sigma \geq 0) \quad (48)$$

Because of the exponential term in the denominator, equation (48) is not in the form of equation (16). Hence, at this point, application of the Mikhailov criterion cannot be justified for neutral systems on the basis of the results in reference 2.

CONCLUDING REMARKS

The valid and invalid application of the first and second formulations of the Mikhailov criterion to linear, time-invariant systems with time delays has been examined. It has been demonstrated by an example that the second formulation of the Mikhailov criterion definitely should not be applied to systems with time delays. The first formulation, however, is directly applicable to delayed systems of the retarded type.

These facts appear to be the basis of the discrepancy in earlier and recent works. The statement that the Mikhailov criterion had limited validity to time-delayed systems in the earlier work was based only on the application of the second formulation to retarded systems whereas recent work is equivalent to the first formulation, which does have valid application for such systems.

At present, application of the Mikhailov criterion to neutral systems has not been justified.

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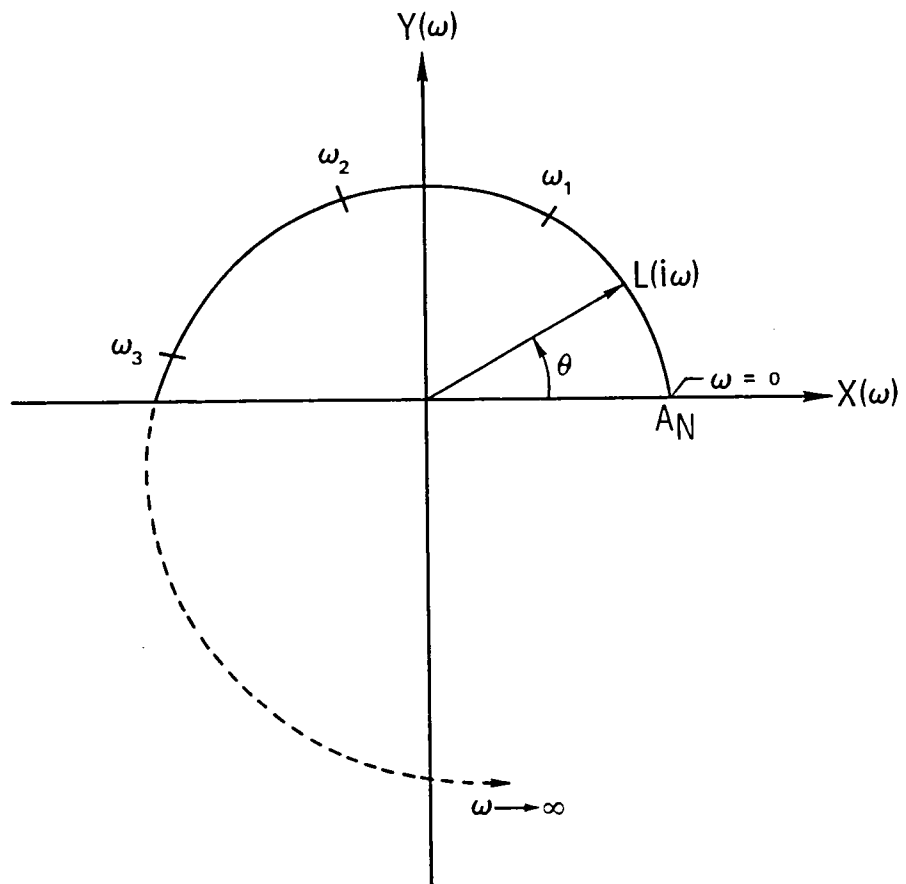
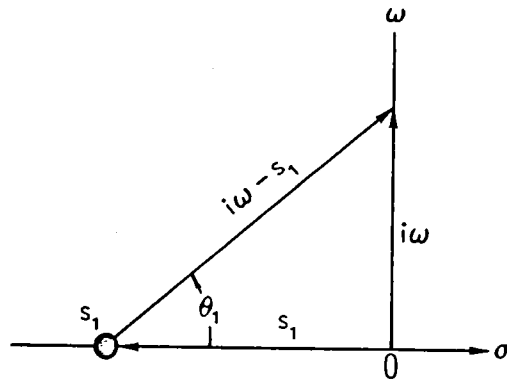
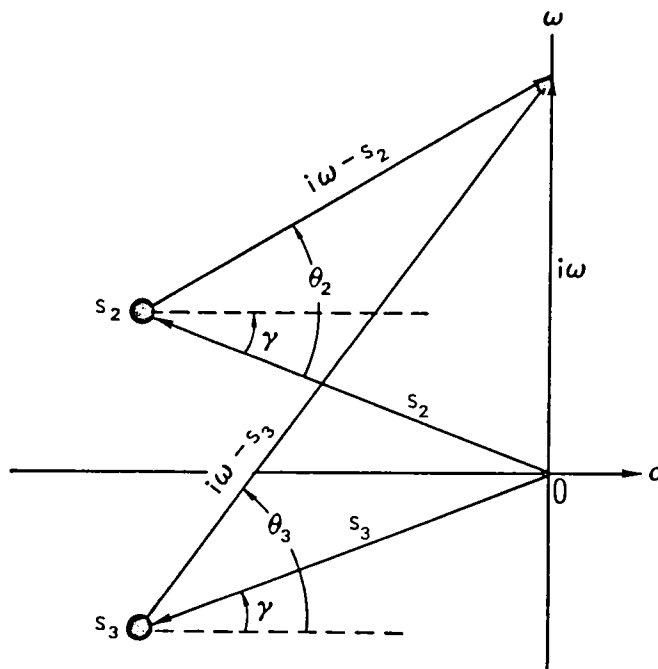


Figure 1.- Mikhailov curve.



(a) Negative real root.



(b) Complex-conjugate roots with negative real parts.

Figure 2.- Complex vector diagrams associated with factors of $L(i\omega)$.

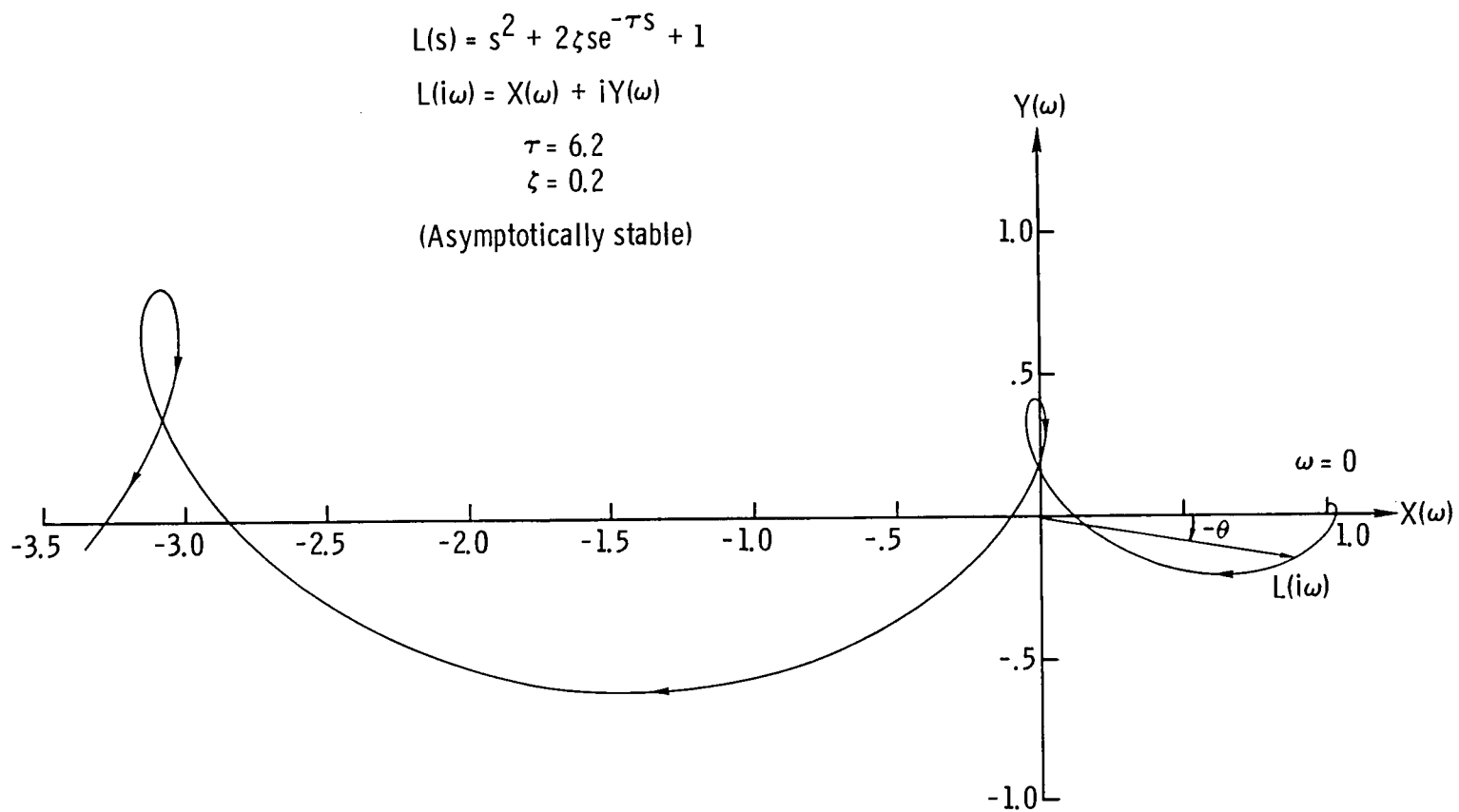


Figure 3.- Mikhailov curve for system with time delay.

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|--|---|-------------------------------|---|--|--|
| 1. Report No. NASA TM-78803 | | 2. Government Accession No. | | 3. Recipient's Catalog No. | |
| 4. Title and Subtitle MIKHAILOV STABILITY CRITERION FOR TIME-DELAYED SYSTEMS | | | | 5. Report Date January 1979 | |
| | | | | 6. Performing Organization Code | |
| 7. Author(s) L. Keith Barker | | | | 8. Performing Organization Report No. L-12281 | |
| 9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665 | | | | 10. Work Unit No. 505-07-33-04 | |
| | | | | 11. Contract or Grant No. | |
| 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 | | | | 13. Type of Report and Period Covered Technical Memorandum | |
| | | | | 14. Sponsoring Agency Code | |
| 15. Supplementary Notes | | | | | |
| 16. Abstract <p>The valid and invalid application of the Mikhailov criterion to linear, time-invariant systems with time delays is discussed. The Mikhailov criterion is a graphical procedure which was initially developed to examine the stability of linear, time-invariant systems with no time delays. For these systems, there are two equivalent formulations of the criterion. Attempting to apply the second formulation when there are time delays in the systems can lead to erroneous results, as shown by an example. However, the first formulation remains valid for time-delayed systems of the retarded type, with the understanding that the Mikhailov curve need not necessarily always rotate in the counterclockwise direction for a stable system. At present, application of the Mikhailov criterion to neutral systems has not been justified.</p> | | | | | |
| 17. Key Words (Suggested by Author(s)) Mikhailov stability criterion Time delay Stability Retarded equations Neutral equations | | | 18. Distribution Statement Unclassified - Unlimited Subject Category 66 | | |
| 19. Security Classif. (of this report) Unclassified | 20. Security Classif. (of this page) Unclassified | 21. No. of Pages 18 | 22. Price* \$ 4.00 | | |

* For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1979

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